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XL. The rotation of the plane of polarization of light by the discharge of a Leyden jar

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problem (48) the displacement-current has gone, so that the existence of H appears to rest merely upon the assumption that moving electrification is true current. But if the plane be not infinite, though large, we shall have (48) nearly true near it, and away from the edges; whilst the displacement-current will be strong near the edges and almost nil where (48) is nearly true.

But in some cases of rotating electrification, there need be no displacement anywhere, except during the setting up of the final state. This brings us to the rather curious question whether there is any difference between the magnetic field of a convection-current produced by the rotation of electrification upon a good nonconductor and upon a good conductor respectively, other than that due to diffusion in the conductor. For in the case of a perfect conductor, it is easy to imagine that the electrification could be at rest, and the moved conductor merely slip past it. Perhaps Professor Rowland's forthcoming experiments on convection-currents may cast some light upon this matter.

December 27, 1888.

XL. *The Rotation of the Plane of Polarization of Light by the Discharge of a Leyden Jar.* *By* DR. OLIVER LODGE*.

THE current produced by the discharge of a Leyden jar is so violent while it lasts, that those phenomena which depend upon the value of a current independently of its duration are well excited by it. Such are the induction of currents, the production of magnetism, and the rotation of the plane of polarization.

Nothing is easier than to wind a quantity of thin gutta-percha-covered wire round a piece of heavy glass, and to witness the bright flashing of a dark field between polarizer and analyser whenver a large Leyden jar is sparked through the coil, the source of light being a paraffin-lamp or gas-flame. The suddenness of the effect suggests, of course erroneously, that it is an illumination caused by the light of the spark which one is looking at.

The fact that the discharge is oscillatory, and that the restoration of light in the dark field is oscillatory too, is proved by the fact that an adjustment of the analyser to one side or the other of complete darkness has just the same effect on the result. It is proved also by the fact that a biquartz

* Communicated by the Physical Society: read March 9, 1889.

exhibits no change in its sensitive tint during the discharge; similarly also Jellett, and other such reversible detectors, would be useless for the purpose: the effect is an irreversible one. Nevertheless some sort of measure of the effect can be made by finding the position of the analyser whereat the brightness of the field suffers no appreciable change at the occurrence of the spark, because in one position the oscillation on one side will darken it just as much as that on the other side brightens it.

It seemed to me possible that if the effect could be rendered pretty considerable a slight *darkening* of the field might be obtained with some adjustments, because some arcs of the sine curve have an average ordinate less than their middle ordinate. But when a fairly bright field winks slightly it is not easy to say whether it winks brighter or darker, and after all I do not know that it much matters.

The main interest in the experiment seemed to me to lie in the evidence it afforded of practical instantaneity in the development of the property in the substance under examination; and in order to try oscillations of much greater frequency than those I first used, I got my assistant, Mr. Robinson, to make a long tube of carbon disulphide with which to repeat the experiment in a sensitive manner.

Now, a most interesting experiment of Villari *, in which he whirled a drum of heavy glass up to 200 revolutions per second between the poles of a magnet, and perceived the electro-optic effect to diminish from 100 revolutions per second upwards, and ultimately nearly cease at high speeds (say 180 per second), had led one to suppose that some distinct time was necessary for the production of the effect—something between $\frac{1}{800}$ and $\frac{1}{400}$ second. But on referring to that most useful summary of Electrical Science, Prof. Chrystal's article in the Encycl. Brit., I found, along with a quotation of Villari's experiment, a statement that Professors Bichat and Blondlot †, of Nancy, had by means of Leyden-jar discharge proved that practically no time was necessary. I accordingly procured a copy of the volume of the *Comptes Rendus* from London, and there found an all too brief account of a most beautiful series of experiments, by which they considered it proved that if any time is required, it is less than the $\frac{1}{30,000}$ second. The skill of these French Professors in optics, and their previous researches in connexion with the Faraday

* Pogg. *Ann.* cxlix. 1873, p. 324. Translated from the *Rendiconti Istit. Lombardo*, ser. 2, vol. iii.

† *Comptes Rendus*, xciv. 1882, p. 1590.

effect in various substances, are well known; and what I have to show to-day is practically nothing more than a repetition of their experiments with the theory worked out.

I take either a bit of heavy glass in its helix, or, for projection, a tube of CS_2 a yard long surrounded by four large helices, each containing about 80 yards of gutta-percha-covered No. 16 wire; and on passing the discharge from a battery of several jars the field flashes out bright, in what may be (if one is looking direct towards the hot lime) quite a dazzling manner.

I have just received a post-card from M. Bichat, in answer to an inquiry, saying that the coil they used was the secondary bobbin of a Rühmkorff coil with a resistance of 5000 ohms. Now, whether this was by accident or by design, it was difficult for them to use a coil more suited to the purpose, or one that would give a larger effect, as I shall show directly. The Leyden jars employed by them were either one or two, about 18 inches high and 6 inches in diameter. We shall find that the effect increases in direct proportion to the capacity of the jars.

To find out whether any time was required for the development of the effect, they made the light coming through the tube illuminate a slit, the spark being made to illuminate another slit close above the first, and then both slits were examined in a rotating mirror. Both were spread out into a discontinuous band, and the serrations of the one agreed as nearly as could be seen with the serrations of the other. Thus proving in a beautiful manner that there was practically no lag of the effect behind its cause, and thereby contradicting the conclusion of Villari*.

Meanwhile I had been doing similar experiments, but with a bobbin of much smaller inductance, and using a still smaller capacity, my object being to find the greatest frequency able to show the effect distinctly. If, for instance, heavy glass or CS_2 was able to follow oscillations of some million per second, there could be no further question but that Villari's conclusion was wrong. I find that the CS_2 is able to show the effect when the rate of alternation is 70,000 per second; and though the short length of heavy glass available does not enable me at present to make quite the same statement for it, I have no

* They also mention an unpublished experiment by MM. Curie and Ledeboer, in which a disk of glass was spun between the poles after the manner of a Foucault disk of copper, so that the path of the light was parallel to the axis of rotation, instead of perpendicular to it as in Villari's drum arrangement, and in that case no diminution was observed. If this experiment is yet published I am ignorant of it.

reason to suppose it in any way inferior. In fact, experiment distinctly suggests that the effect is practically instantaneous; but as to the degree of instantaneity I shall be able to make a more exact numerical statement later on.

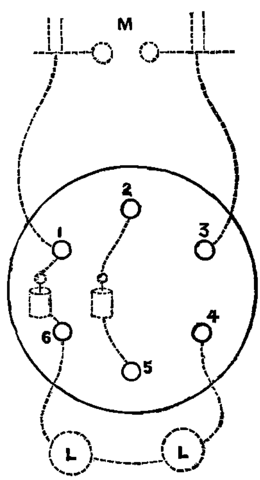
It may be of interest to the Physical Society to have the oscillatory character of the restored light demonstrated, and there is no difficulty in the experiment. One sets the analyser to as near darkness as possible, one receives the trace of residual light upon a rotating mirror, by which it is spread out into a faint band; and on then sending sparks through the coil of wire round the tube of CS_2, the band brightens and presents a distinctly beaded appearance at every spark.

Rotating the analyser a little, every alternate bead grows fainter, while the other alternate ones brighten, thus proving most directly the oscillatory character of the light and of the Leyden-jar discharge.

Still more feasible is it to spread out the light of the spark itself into a serrated band, but in making this visible to an audience it is well to save time by exciting the jars with a large induction-coil instead of a Wimshurst machine, because not only can sparks be thus got much more frequently, but each spark is multiple—the jars filling and overflowing several times during the one coil-discharge. The multiple or intermittent spark is analysed by the revolving mirror into a number of serrated bands one after the other, and while one band is in the field of view of one part of the audience another band may be visible to another.

Although when a suitable circuit is employed the analysis of the spark in a mirror such as is used for manometric flames rotating not more than three or four times a second is easy, yet with ordinary discharging circuits I have used small mirrors spinning 200 times a second and failed to see any certain trace of oscillation; while, as is well known, Wheatstone used mirrors rotating 800 times a second, and got the image-spark only barely elongated: not in the least serrated. It may be worth while, therefore, to state the kind of circuit which I have recently employed. The capacity consists of a couple of condensers built up in the laboratory with double thicknesses of window-glass alternating with tinfoil, and the whole flooded in paraffin till it is a solid mass in a box of teak weighing a couple of hundredweight. These condensers stand a considerable length of spark, 3 or 4 inches for instance, but their strength is not in the least called out in the experiments now related. Their capacity when joined up parallel is ·048 microfarad. These are often supplemented by a battery of ordinary Leyden jars and by single jars, which altogether raise the capacity to ·074 microfarad or 670 metres. To vary the capacity in the ratio 1, 2, 4, I use a switch consisting of 6 glass pillars, each with a bit of close-fitting glass tube cemented on the top to serve as a mercury cup. The leading wires are held in the mercury cups by indiarubber umbrella-rings round the glass pillars; and insulated wire bridges easily make connexion between one pillar and the next.

In the annexed figure dotted lines indicate sufficiently the permanent connexions. M being machine and discharger, L being circuit and coils. The movable connexions are made in pairs as follows:—



For series or cascade, connect 2, 3; 4, 5.

For one condenser only, connect 3, 4.

For both condensers in parallel, connect 1, 2; 3, 4; 5, 6.

As regards circuit, one of the coils I use is a hank of 440 yards of thickly covered No. 16 G. P. wire as it came from the maker, with a self-induction of ·048 secohm and a resistance of 3·75 ohms.

The coils on the CS_2 tube consist of the same kind of wire; they have a resistance rather greater than the above, and a combined self-induction of about ·008 secohm.

A number of gigantic electric-light cables have been inserted in the circuit at different times, one on a bobbin like two cart-wheels being kindly lent by the Engineers of the Liverpool Electric Supply Company; but the easiest way of getting very large self-induction is to use No. 20 or No. 22 G. P. or I. R. wire not too thickly covered. I have recently obtained a large coil wound so as to give maximum self-induction which I estimate as 7 or 8 secohms; I have not yet used this, but I frequently use a number of coils of No. 21 wire packed together as close as is easily possible, and making a total self-induction of 1 secohm or ten thousand kilometres.

Without going into further details, I may say that the observed frequency of the oscillations, as estimated from the appearance of the serrations in the revolving mirror, or by the pitch of the musical note accompanying the spark, agrees very respectably with the frequency as calculated from

$$\frac{3\times 10^{10}}{2\pi\sqrt{\left(\frac{L}{\mu}\cdot\frac{S}{K}\right)}}.$$

Returning now to the magneto-optic effect of the Leyden-jar discharge, I make out its theory to be as follows:—

The current at any instant during the discharge is

$$C=\frac{V_0}{pL}e^{-mt}\sin pt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{where } m=\frac{R}{2L} \text{ and where } m^2+p^2=\frac{1}{SL}.$$

The difference of magnetic potential between the two ends of a rod of length l surrounded by a very long solenoid of n_1 turns of wire on unit length of it is

$$4\pi\mu C n_1 l.$$

The Verdet constant for carbon disulphide, or the rotation of the plane of polarization per unit difference of magnetic potential is, according to the determination of Lord Rayleigh, ·04202 minute for a temperature of 18° Centigrade, and for sodium light. Calling this k, we have the rotation effected by the current

$$\theta=4\pi k n_1 l\mu C. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the case of the heavy glass experiment a helix longer than the stalk of glass is employed; but in the case of the long tube of CS_2 and four helices there will be some correction necessary for the ends of the helices. This correction is, however, well understood, and there is no special need to introduce it at present.

Now the total amplitude of the light falling upon the analyser being a (this amplitude being all stopped if the analyser is set to darkness, and all transmitted if it be turned 90°), the component amplitude which will at any instant get through the analyser set to darkness, when the plane of polarization has been rotated through an angle θ, is

$$a\sin\theta;$$

and the amount of light which gets through during the time τ during which an impression is capable of being accumulated on the retina, *i.e.* before it has begun to die away as fast as it is produced, is

$$\int_0^\tau (a\sin\theta)^2 dt.$$

The amount of light which would get through in the same time if the analyser were set to maximum brightness would be

$$a^2\tau.$$

These expressions, then, furnish the measure of the respective effects upon the retina; and so we have

$$\frac{\text{apparent brightness restored by the spark}}{\text{maximum brightness possible}}=\frac{\int_0^\tau a^2 \sin^2\theta\, dt}{a^2\tau}.$$

Now inasmuch as τ is comparable to the time of persistence of retinal impression, being perhaps equal to it, and since this time is greatly longer than any ordinary duration of a spark-discharge, the above ratio of the relative brightnesses reduces, for practical purposes, to

$$\mathrm{B} = \text{relative brightness} = \frac{1}{\tau}\int_0^\infty \sin^2\theta\, dt, \quad . \quad . \quad (3)$$

and this is what the eye will observe.

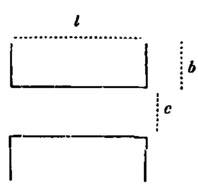
Referring back to equations (1) and (2) we have the means of determining this quantity. I do not know how to do it completely, but for the case when θ is moderately small the integral is easy, viz.:—

$$\mathrm{B}=16\pi^2k^2\mu^2n^2\frac{\mathrm{V}_0^2}{p^2\mathrm{L}^2\tau}\int_0^\infty e^{-2mt}\sin^2 pt\, dt$$

$$=160\,k^2\mu^2n^2\,\frac{\frac{1}{2}\mathrm{SV}_0^2}{\mathrm{R}\tau}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The effect thus depends directly on the square of the total number of turns of wire employed, directly on the energy of the static charge used, and inversely on the resistance of the circuit.

To find the best size of wire to wind on a bobbin of given size, for the purpose, one can write down the value of n^2/R; given the length of the bobbin as l, its depth of winding-space b, the diameter of its empty core c. Call the radius of the uncovered wire used ρ, and its radius when covered ρ'.



First, supposing no appreciable resistance in the rest of the circuit, it comes out

$$\frac{n^2}{\mathrm{R}}=\frac{lb}{4(c+b)}\left(\frac{\rho}{\rho'}\right)^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which means that the size of the wire does not matter, but that it is important to keep the covering thin. Only then unfortunately the discharge is liable to burst the covering. In my helices $\rho'=3\rho$, which is rather excessive, because it reduces the visible effect to one ninth of what it might be if no covering were used.

It is impossible to press this to its natural consequence of using only one turn of thick sheet copper, because the resistance of the rest of the circuit has been neglected. Taking it into account, as r, the relation becomes

$$\frac{n^2}{\mathrm{R}}=\frac{\dfrac{lb}{4(c+b)}\cdot\left(\dfrac{\rho}{\rho'}\right)^2}{1+\dfrac{4r}{lb(c+b)}(\rho\,\rho')^2}, \quad . \quad . \quad . \quad . \quad (6)$$

which contains the product of the sizes of covered and uncovered wire as well as the ratio; and this product occurs in the term containing r, the external resistance. Hence, to keep this term small, it is desirable to use wire thin enough to throw the major part of the resistance into the bobbin; and there is no limit to the thinness of the wire that may be advantageously employed, until the thickness of the covering bears too high a ratio to the whole. And inasmuch as insulation thickness may be more judiciously distributed between layers than between consecutive turns, it is obvious how extremely suitable a coil for the purpose is the secondary of a Rühmkorff.

I said that I did not know how to evaluate the complete integral involved in (3) when θ is not small; but, as usual, I sent the problem to my brother, and he speedily reduced it to a form equivalent to this:—

$$\mathrm{B}=\frac{\mathrm{L}}{\mathrm{R}\tau}\int_0^{\mathrm{A}}\frac{1-\mathrm{J}_0(x)}{x}\,.\,dx, \quad . \quad . \quad . \quad . \quad (7)$$

where

$$\mathrm{A}=8\pi kn\mu\mathrm{V}_0\sqrt{\left(\frac{\mathrm{S}}{\mathrm{L}}\right)}.$$

I have asked him to write a short appendix to this paper.

Returning now to consider the meaning of these equations, and attempting a numerical estimate of what to expect in practice, we shall find that though the instantaneous rotation expressed by (2) is enormous, being quite possibly 60 or even 180 degrees, yet the restoration of light expressed by (4) is but feeble, and only some ten thousandth part or so of what could be gained by rotating the Nicol. This explains why it is fairly easy to analyse the restored light into a beaded band, because then one gets the effect of the separate oscillations; and they are very bright though very momentary.

Looking at the formula (4), it is clear that to get large effects it is desirable to use a large battery of jars, and to charge to a high potential, only that one is afraid of breaking down the coil. Large capacity and a great number of turns of wire are the safest ways of increasing the effects.

One sees also that extra self-induction in the circuit does neither good nor harm to the resultant effect. It diminishes the effect of each oscillation, but it prolongs the time during which they last. And it is the total "*action*" of the series of decaying swings which the eye perceives. One sees that extra resistance in the circuit is wholly bad.

As to the cause of what must now be regarded provisionally as the erroneous conclusion of Villari I see no reason to doubt his experiments, though he does not give sufficient details to enable one to arrive at a perfectly satisfactory judgment on all points; it is practically certain that he did get a much diminished effect on spinning the flint-glass drum between the poles of the magnet and sending the light along successive diameters of the drum.

But the cause of this I venture to suggest is possibly to be found in the state of strain into which the glass will be thrown by centrifugal force. It may be said that, if so, the light ought to have been similarly affected even when no magnetic field was employed; and Villari expressly says that this was not the case.

But then it is to be noticed that, when no magnetic rotation is attempted, the aspect of the plane of polarization to the stress remains constant throughout the journey; and if light happens to enter with no component modifiable by the stress, it will go out in the same condition. Whereas when rotation has taken place inside the glass this constancy of aspect is destroyed, and the light on exit has a different component modifiable by the strain to what it had on entrance.

I do not profess to be able to give a coherent account of how this cause shall give rise to a reduction in the rotation instead of to an elliptical polarization. But then neither am I able to extract from Villari's account of his experiments any assurance that some elliptic polarization was not produced, and that the reduction of the rotation of the polarized plane was anything more than a mixture of small effects not easily analysable nor precisely defined.

It is in any case a most interesting experiment, and should be repeated so as to really get at the bottom of the cause of the observed phenomenon. There are many other experiments on whirling glass which may likewise be made.

APPENDIX, *by* ALFRED LODGE, M.A., *Coopers Hill, Staines.*

The value of

$$\int_0^\infty \sin^2\theta\, dt, \text{ where } \theta = \mathrm{A}e^{-mt}\sin nt, \text{ and } \mathrm{C} = \frac{\mathrm{A}n}{\sqrt{m^2+n^2}},$$

is $$\frac{1}{2m}\left(\tfrac{1}{2}.\mathrm{C}^2 - \tfrac{1}{4}.\frac{\mathrm{C}^4}{(\underline{|2})^2} + \tfrac{1}{6}.\frac{\mathrm{C}^6}{(\underline{|3})^2} - \tfrac{1}{8}.\frac{\mathrm{C}^8}{(\underline{|4})^2} + \dots\right).$$

For

$$\int_0^\infty \sin^2\theta\, dt = \tfrac{1}{2}\int_0^\infty (1-\cos 2\theta)\, dt$$

$$= \tfrac{1}{2}\int_0^\infty \left(\frac{(2\theta)^2}{\underline{|2}} - \frac{(2\theta)^4}{\underline{|4}} + \dots\right) dt.$$

The general term of this series is

$$\frac{2^{2p}\mathrm{A}^{2p}}{\underline{|2p}}\int_0^\infty e^{-2mpt}\sin^{2p} nt\, dt,$$

and can be integrated by successive reduction.

Let u_{2p} denote

$$\int_0^\infty e^{-2mpt}\sin^{2p} nt\, dt;$$

then, integrating by parts,

$$u_{2p} = \frac{-e^{-2mpt}}{2mp}.\sin^{2p} nt\Big]_0^\infty + \int_0^\infty \frac{e^{-2mpt}}{2mp}.2np\sin^{2p-1} nt.\cos nt\, dt$$

$$= 0 + \frac{n}{m}\int_0^\infty e^{-2mpt}\sin^{2p-1} nt.\cos nt\, dt;$$

and again integrating by parts,

$$= -\frac{n}{m}\frac{e^{-2mpt}}{2mp}.\sin^{2p-1} nt.\cos nt\Big]_0^\infty$$

$$+\frac{n}{m}\int_0^\infty \frac{e^{-2mpt}}{2mp}\left((2p-1)\sin^{2p-2} nt\cos^2 nt - \sin^{2p} nt\right).n.dt$$

$$= 0 + \frac{n^2}{2m^2p}\int_0^\infty e^{-2mpt}\left((2p-1)\sin^{2p-2} nt - 2p.\sin^{2p} nt\right) dt$$

$$= \frac{n^2}{m^2}.\frac{2p-1}{2p}.u_{2p-2} - \frac{n^2}{m^2}.u_{2p}.$$

$$\therefore \quad \left(1+\frac{n^2}{m^2}\right)u_{2p}=\frac{n^2}{m^2}\cdot\frac{2p-1}{2p}u_{2p-2};$$

$$\therefore \quad u_{2p}=\frac{n^2}{m^2+n^2}\cdot\frac{2p-1}{2p}\cdot u_{2p-2}$$

$$=\left[\frac{n^2}{m^2+n^2}\right]^p\cdot\frac{2p-1}{2p}\cdot\frac{2p-3}{2p-2}\cdots\tfrac{3}{4}\cdot\tfrac{1}{2}\int_0^\infty e^{-2mpt}\,dt$$

$$=\frac{1}{2mp}\left[\frac{n^2}{m^2+n^2}\right]^p\cdot\frac{(2p-1)(2p-3)\ldots 3.1}{2p(2p-2)\ \ldots\ 4.2}=\frac{\lfloor 2p}{2^{2p}\lfloor p\lfloor p};$$

$$\therefore \quad \frac{(2\mathrm{A})^{2p}}{\lfloor 2p}u_{2p}=\frac{\mathrm{A}^{2p}}{2mp(\lfloor p)^2}\cdot\left[\frac{n^2}{m^2+n^2}\right]^p=\frac{\mathrm{C}^{2p}}{2mp(\lfloor p)^2};$$

$$\therefore \quad \int_0^\infty \sin^2\theta\,dt=\frac{1}{2m}\left(\tfrac{1}{2}.\mathrm{C}^2-\tfrac{1}{4}.\frac{\mathrm{C}^4}{(\lfloor 2)^2}+\tfrac{1}{6}.\frac{\mathrm{C}^6}{(\lfloor 3)^2}-\ldots\right).$$

Q. E. D.

N.B.—The symbols used for the constants throughout this Appendix have no connexion with those in the main paper, except that m is the same in both. The value of the C of the Appendix, expressed in terms of the symbols used in the paper, is

$$4\pi kn\mu\mathrm{V}\omega\sqrt{\mathrm{S}\div\mathrm{L}}.$$

XLI. *On the Use of* Lissajous' *Figures to determine a Rate of Rotation, and of a Morse Receiver to Measure the Periodic Time of a Reed or Tuning-fork. By* Prof. J. Viriamu Jones, *M.A.**

It is sometimes of importance to determine with great accuracy the angular velocity of a rotating body at a given instant. For instance, in measuring an electrical resistance in absolute measure by the British Association method or the method of Lorenz, the rate of rotation of the revolving coil or disk must be known with full accuracy at the time when the reading of a galvanometer-needle is taken.

The method I have to bring before the Physical Society in this note consists in obtaining, by means of Lissajous' figures, equality of period between the rotating body and a reed maintained in vibration electrically, and then subsequently determining the vibration-period of the reed.

* Communicated by the Physical Society: read March 23, 1889.